# The Effects of Autocorrelation on the Curve-of-Factors Growth Model

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Abstract

This simulation study examined the performance of the curve-of-factors model (COFM) when autocorrelation and growth processes were present in the first-level factor structure. In addition to the standard curve-of factors growth model, two new models were examined: one COFM that included a first-order autoregressive autocorrelation parameter, and a second model that included first-order autoregressive and moving average autocorrelation parameters. The results indicated that the estimates of the overall trend in the data were accurate regardless of model specification across most conditions. Variance components estimates were biased across many conditions but improved as sample size and series length increased. In general, the two models that incorporated autocorrelation parameters performed well when sample size and series length were large. The COFM had the best overall performance.

The Effects of Autocorrelation on the Curve-of-factors Growth Model

The proliferation of longitudinal panel data sets (i.e., data that tracks the same set of subjects across repeated observations) tracking children, adults, communities, organizations, etc., has increased interest in statistical models that measure change across time. There are a variety of statistical techniques available with which to model change across time. Some commonly used models include repeated measures multivariate analysis of variance (MANOVA), autoregressive or quasi-simplex models, multilevel models, and latent growth models. These models differ in how well they capture the research questions of interest and how well the assumptions underlying the models match the empirical data (Curran & Bollen, 2001). The decision as to which model will best fit the data is critical, and it is rarely clear cut.

Two growth models from the structural equation modeling (SEM) framework that have received considerable attention in the social sciences are the autoregressive quasi-simplex model and the latent growth curve (sometimes called latent trajectory) model (Bollen & Curran, 2004; Curran & Bollen, 2001). Traditionally researchers have attempted to identify the conditions under which the growth curve and autoregressive approaches do or do not fit empirical longitudinal data (Bast & Reitsma, 1997; Curran, 2000; Kenny & Campbell, 1989; Marsh, 1993; and Rogosa & Willett, 1985). This comparative approach has inadvertently fostered an either/or perspective with regard to modeling longitudinal data. If the focus is on differences among individual growth trajectories, then latent growth models are considered to be more appropriate. In contrast, if the data are assumed to be a series of correlated events whereupon the initial value of interest determines the growth trajectory for each participant, then autoregressive models are considered to be more appropriate.

Latent growth models allow each participant to have an individual growth trajectory which does not necessarily depend on the magnitude of the initial measurement value. Autocorrelation is considered to be a nuisance variable. Failure to model autocorrelation when it is present in the data has been shown to bias latent growth curve and multilevel growth curve model parameters of interest (Ferron, Dailey, & Yi, 2002; Kwok, West, & Green, 2007; Murphy & Pituch, 2009; Sivo, Fan, & Witta, 2005).

By contrast, autoregressive models specify recently measured variables as a function of earlier measurements. The variables are considered to be correlated across time, with variables closer together in time more highly correlated than those further apart. For example, in a first- order autoregressive model, the first and second observations would be more highly correlated than the first and third observations. The rank order of the individuals under this model remains stable across time.

A third type of structural equation model that has attracted recent interest (e.g., Leite, 2007) is the COFM (McArdle, 1988; Tisak & Meredith, 1990), which is sometimes called a second-order latent growth model (Hancock, Kuo, & Lawrence, 2001; Sayer & Cumsille, 2001). In contrast with the latent growth model which models growth as a first-order factor, the COFM models growth as a second-order factor. The first-order factors in a COFM are indicated by multiple manifest variables that are measured repeatedly across time, and the second-order factors indicate the initial factor level of interest and the shape of the growth trajectory. Two major advantages of the COFM are: 1) the factors can be considered to be "true" scores because the measurement error is modeled, and 2) measurement invariance across subgroups can be evaluated. By contrast, the growth curve models mentioned previously assume that the measurements are invariant across subgroups and no measurement error exists (Hancock et al., 2001; Leite, 2007). The effects of unmodeled autocorrelation within the COFM have yet to be studied.

This study revolved around two related goals. First, the effects of unmodeled or mismodeled autocorrelation within the COFM were examined. Second, the COFM was combined with two different autocorrelation processes, a first-order autoregressive process [AR(1)] and a first-order autoregressive-moving average process [ARMA(1, 1)] in an attempt to integrate the models. Based on previous research into the effects of autocorrelation on growth models, we expect unmodeled autocorrelation within the curve-of-factors framework to result in biased parameter estimates, in particular among the variance components. We expect better performance from the models that combine autoregressive and growth parameters. The next section of the paper describes the three models that were examined in this study.

## Unconditional Curve-of-factors Model

First-order latent growth models model change over time in measured variables (e.g., Bollen, 2004; McArdle, 1988; Meredith & Tisak, 1990; Singer & Willett, 2001; Stoolmiller, 1995). By contrast, the COFM models the outcome measures to be indicators of a latent construct, which, when measured repeatedly, forms the first-order factor structure. Thus, this first-order common factor portion of the COFM consists of latent constructs, manifest indicators, and measurement error. In theory, by accounting for measurement error the COFM provides an error-free construct for growth modeling (Hancock et al., 2001).

For change being measured across $t$ time points, let $\eta_j$ represent a latent construct indicated at time $j$ by $k$ measured variables $Y_{ij}$ ($i = 1,\ldots, k$). The measurement portion of the COFM can be expressed as follows:

$$\boldsymbol{y} = \boldsymbol{\tau} + \boldsymbol{\Lambda\eta} + \boldsymbol{\varepsilon}, \tag{1}$$

where $\boldsymbol{y}$ is a vector containing $t$ sets of $Y_{ij}$, $\boldsymbol{\tau}$ is a vector of indicator intercepts, $\boldsymbol{\Lambda}$ is a $kt$ x $t$ matrix specifying the factor loadings relating each $\eta_j$ latent construct to its indicator variables, $\boldsymbol{\eta}$ is a $t$ x 1 vector of the $\eta_j$ latent constructs, and $\boldsymbol{\varepsilon}$ is a $kt$ x 1 vector of random normal errors (i.e., measurement error).

The COFM is sometimes referred to as a second-order latent growth model (Sayer & Cumsille, 2001) because the growth parameters are modeled as second-order factors. The second-order portion of the structural model specifies the growth parameters, i.e., the level and the shape, of the first-order $\eta_j$ constructs. The structural portion of the COFM can be expressed as (Hancock et al., 2001):

$$\boldsymbol{\eta} = \boldsymbol{\Gamma\xi} + \boldsymbol{\zeta}, \tag{2}$$

where $\boldsymbol{\eta}$ is a vector of the first-order $\eta_j$ constructs, $\boldsymbol{\Gamma}$ is a $t$ x 2 (for the linear model), matrix of second-order factor loadings reflecting the growth pattern underlying the $\eta_j$ constructs, $\boldsymbol{\xi}$ is a 2 x 1 vector of second-order factors capturing the level (i.e., $\alpha$) and shape (i.e., $\beta$) parameters of the first-order factors, and $\boldsymbol{\zeta}$ is a $t$ x 1 vector of random normal disturbances. The loadings of the latent growth part of the COFM (i.e., the $\boldsymbol{\Gamma}$ matrix) can be fixed to values that reflect a specific hypothesis about the shape of the growth, as in first-order latent growth models. The loadings of the latent growth parameters can also be estimated freely from the data rather than being specified by the researcher.

The second-order latent growth parameters (i.e., the level and shape) are modeled as:

$$\boldsymbol{\xi} = \boldsymbol{\mu} + \boldsymbol{\zeta}_{\alpha\beta}, \quad (3)$$

where $\xi$ is a vector containing the level and shape parameters for each individual, $\boldsymbol{\mu}$ is the vector of the level and shape means, and $\boldsymbol{\zeta}_{\alpha\beta}$ is a 2 x 1 vector of random normal disturbances of the level and shape.

*Autocorrelation in the Curve-of-Factors Model*

The first-order factor structure in Equation 1 consists of $\eta_j$ factor scores, each of which is indicated by multiple manifest variables. The COFM is designed to summarize factor score changes within and between participants across time. There is, however, an alternative modeling technique that may be more appropriate when the factor scores are correlated, the individual trajectory depends on the initial measurement value, and the change between participants is minimal.

Guttman (1954) noted that repeated measures of psychological tests formed a simplex, whereby measures that were closer in time correlated more highly than more distant measures. Repeated measures that are correlated can be modeled for two stochastic processes: autoregressive and moving average (Box & Jenkins, 1976). The use of structural equation modeling techniques to model autoregressive and moving average processes is well established, and the interested reader is referred to a number of excellent sources (e.g., Cook & Campbell, 1979; Jöreskog, 1978, 1979; McArdle & Aber, 1990; Rovine & Molenaar, 2005; Sivo, 2001; van Buuren, 1997) This section of the paper will present a model that combines elements of a first-order autoregressive moving average [ARMA(1, 1)] quasi-simplex model with the second-order growth factors of the COFM.

The addition of an ARMA(1, 1) process to the first-order factor structure of the COFM can be presented as,

$$\boldsymbol{\eta} = \boldsymbol{\Gamma\xi} + \mathbf{B}\boldsymbol{\eta} + \boldsymbol{\zeta}. \tag{4}$$

In Equation 4, as in Equation 2, $\boldsymbol{\eta}$ represents the vector of $\eta_j$ first-order latent constructs. $\mathbf{B}$ represents a $t$ x $t$ matrix of regression coefficients that specify the relationship between adjacent latent factors measured on occasions $j - 1$ and $j$, and $\boldsymbol{\zeta}$ represents a vector of disturbances modeled as latent error factors for each occasion.

Although the model described by Equation 4 may appear to be a simple combination of the COFM and an ARMA(1, 1) quasi-simplex model, there is a mitigating circumstance. It is possible that the autoregressive moving average function extends prior to the first wave of data, meaning the first wave of data would be dependent on a previous wave of data. A simple way to avoid the complications associated with this implication is to treat the first observation as predetermined, as demonstrated under the first-order autoregressive latent trajectory model developed by Bollen and Curran (2004). The predetermined first observation can be modeled to correlate with the second order growth parameters (e.g., $\alpha_i$ and $\beta_i$).

The covariance structure of this combined model includes the covariances of the measurement model, the covariances of the first-order structural model, and the covariances of the second-order structural model. The variance-covariance equation for the common factor portion of the model can be expressed as

$$\boldsymbol{\Sigma}_{\mathbf{yy}} = \boldsymbol{\Lambda}(\boldsymbol{I} - \mathbf{B})^{-1}\boldsymbol{\Omega}(\boldsymbol{I} - \mathbf{B})'^{-1}\boldsymbol{\Lambda}' + \boldsymbol{\Theta}, \tag{6}$$

where $\mathbf{\Sigma_{yy}}$ is the variance-covariance matrix of the $Y_{ij}$ indicators, $\mathbf{\Lambda}$ is the matrix of factor loadings described in Equation 1, $\mathbf{B}$ is the matrix of structural regression coefficients described in Equations 4 and 5, $\mathbf{\Omega}$ is the covariance matrix of the $\eta_j$ first-order latent factors, and $\mathbf{\Theta}$ is a covariance matrix of the $\varepsilon_{ij}$ measurement errors of the items.

The implied covariance matrix for the latent-growth portion of the model is

$$\mathbf{\Omega} = \mathbf{\Gamma\Phi\Gamma}' + \mathbf{\Psi}, \tag{7}$$

where $\mathbf{\Gamma}$ is the matrix of loadings for the level and shape described in Equation 4, $\mathbf{\Phi}$ is a 2 x 2 (for a linear model) covariance matrix of the level and shape factors, and $\mathbf{\Psi}$ is the covariance structure of the first-order latent factors described in Equation 5. Substituting Equation 7 into Equation 6 results in the model-implied covariance matrix,

$$\mathbf{\Sigma_{yy}} = \mathbf{\Lambda}(\boldsymbol{I} - \mathbf{B})^{-1}(\mathbf{\Gamma\Phi\Gamma}' + \mathbf{\Psi})(\boldsymbol{I} - \mathbf{B})^{'-1}\mathbf{\Lambda}' + \mathbf{\Theta}, \tag{8}$$

The $\mathbf{B}$ and $\mathbf{\Psi}$ matrices in Equation 8 are the specific matrices that are mismodeled under the COFM when an AR(1) or ARMA(1, 1) autocorrelation process is present in the first-order factor structure. However, Hamaker (2005) demonstrated that when the autoregressive parameter φ does not vary across time and $|\varphi| < 1$ , latent growth models with autoregressive relationships between the observed variables and latent growth models with autoregressive relationships between the disturbances are algebraically equivalent. In the Appendix, we extend Hamaker's proof to the case where moving average relationships are included in the model, provided the moving average parameter θ does not vary across time and $|\theta| < 1$.

As a result, estimates of the fixed effects and random effects of the overall linear trend in the data under the COFM are functionally related to estimates under the COFM models with AR(1) and ARMA(1, 1) parameters. Thus, because these models are functionally related, it is expected that the parameter estimates of interest will be successfully reproduced under all three models.

## Method

*Conditions and Parameters*

This simulation study modified a SAS macro developed by Fan, Felsöványi, Sivo, and Keenan for generating and estimating multivariate data (2001). Monte Carlo methods were used to generate longitudinal data with a single set of growth parameters data using SAS/IML. The data were then analyzed using the SAS PROC CALIS procedure. The first-order factors of the COFM used to generate data for this study were indicated by four observed variables that were measured repeatedly across equally spaced time points as presented in Equation 1.

The second-order factors modeled the level and shape of the first-order latent factors as described in Equation 2. For each condition, the population means of the overall linear trend in the data were set to 0 and 0.5 respectively. Note that the population intercept and slope means of the overall linear trend in the data are related to the level mean $\mu_\alpha$, the shape mean $\mu_\beta$, and the autoregressive parameter $\varphi$ in the following manner (Hamaker, 2005):

$$\mu_\delta = \mu_\alpha(1-\varphi)^{-1} - \varphi\mu_\beta(1-\varphi)^{-2} \quad (9)$$

$$\mu_\gamma = \mu_\beta(1-\varphi)^{-1} \quad (10)$$

where $\mu_\delta$ is the mean of the intercept, and $\mu_\gamma$ is the mean of the slope.

The variances of the level and shape parameters were set to 0.5 and 0.1, respectively, which are consistent with the parameter values used in previous simulation studies (e.g., Leite, 2007; Sivo et al., 2005). The covariance between the level and shape was set to 0. Note that the variance of intercept, slope, and the covariance between them are also functionally related to the level and shape variances and covariance (i.e., $\sigma_\alpha^2$, $\sigma_\beta^2$, and $\sigma_{\alpha\beta}$ respectively), and the autoregressive parameter φ in the following manner (see Appendix):

$$\sigma_\delta^2 = \sigma_\alpha^2(1-\varphi)^{-2} + \varphi^2\sigma_\beta^2(1-\varphi)^{-4} - 2\varphi\sigma_{\alpha\beta}(1-\varphi)^{-3} \quad (11)$$

$$\sigma_\gamma^2 = \sigma_\beta^2(1-\varphi)^{-2} \quad (12)$$

$$\sigma_{\delta\gamma} = \sigma_{\alpha\beta}(1-\varphi)^{-2} - \varphi\sigma_\beta^2(1-\varphi)^{-3}, \quad (13)$$

where $\sigma_\delta^2$ is the intercept variance, $\sigma_\gamma^2$ is the slope variance, and $\sigma_{\delta\gamma}$ is the covariance between the intercept and slope. The parameters of the measurement model were simulated to be identical across time, meaning strict factorial invariance was generated. The item intercepts all were generated to be 0, the factor loadings all were generated to be 1, and the error variances for all items were generated to be 1.

Four factors were systematically varied in this study. First, the values and parameters of the autocorrelation process were generated to model an AR(1), ARMA(1, 1), or control (i.e., no autocorrelation) process. Second, the sample sizes were simulated to be 100, 200, 500, or 1,000. Third, the measurement occasion series length was varied to be either 5 or 8 simulated measurement occasions. Fourth, the COFM utilized to analyze the data was specified to be either: 1) a COFM; 2) a COFM with an AR(1) parameter added to the first-order factor structure; or 3) a COFM with two ARMA(1, 1) parameters added to the first-order factor structure.

For the ARMA(1, 1) process, the autocorrelations of the AR(1) portion of the process φ took on two different values, one correlation with moderate magnitude, .5, and one correlation with a large magnitude, .8. In addition, the correlations of the MA(1) process θ also took on two values; a correlation .3 was paired with the large magnitude correlation of the AR(1) process (i.e., .8), and a correlation of -.3 was paired with the moderate magnitude correlation of the AR(1) process (i.e., .5). These ARMA(1, 1) parameter values were selected and paired because in combination they model autocorrelation with a starting value between .6 and .7 which decays more slowly and more quickly, respectively, than a pure AR(1) process.

By setting θ = 0, two AR(1) processes were generated, a large autocorrelation where φ = .8 and a small to moderate autocorrelation where φ = .3. Finally, by constraining both φ and θ to equal zero, a set of data without autocorrelation was generated as a control model. All parameter values are within the ranges of values commonly studied in previous simulations of AR(1) and ARMA(1, 1) data (Ferron et al., 2002; Hamaker, Dolan, & Molenaar, 2002; Murphy & Pituch, 2009; Sivo et al., 2005; Sivo & Willson, 2000).

The specification of the estimating COFM, which was treated as a repeated measures design factor, consisted of three levels: a pure COFM, a COFM integrated with an AR(1) autoregressive parameter, and a COFM integrated with two ARMA(1, 1) parameters in the structural model. This factor was crossed with all study design factors. Thus, the study design had 120 cells. To summarize, the data were generated as a 5 (autocorrelation structure) x 4 (sample size) x 2 (series length) factorial design. For each cell, a total of 1,000 data sets were generated resulting in a total of 40,000 data sets, each of which was analyzed using the three COFM specifications.

*Data Generation*

The implied population covariance matrix was obtained by inserting the covariance population values for each study condition into Equation 8 and computed using SAS IML programming language (SAS Institute, 2005). Once the population covariance matrix and mean vector were established, 1,000 multivariate normal random draws were generated for each condition.

Note that it is possible that autocorrelation functions extend prior to the first wave of data, meaning the first wave of data would be dependent on a previous wave of data (i.e., to predict $y_1$ in an AR(1) model, we would need $y_0$). This dependency on a previous wave of data implies that the process must be started up. One method of starting up an AR(1) process when generating data uses the extra parameter $\sigma_\eta^2$, which can be interpreted as the variance of the series and defined as $\sigma_\eta^2 = \frac{\sigma_\zeta^2}{(1-\varphi)^2}$ (Hamaker, Dolan, & Molenaar, 2003). For example, for 5 measurement occasions we generated the AR(1) data by specifying the **B** and **Ψ** matrices in Equation 8 as

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \varphi & 0 & 0 & 0 & 0 \\ 0 & \varphi & 0 & 0 & 0 \\ 0 & 0 & \varphi & 0 & 0 \\ 0 & 0 & 0 & \varphi & 0 \end{bmatrix}; \ \mathbf{\Psi} = \begin{bmatrix} \sigma_\eta^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_\zeta^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_\zeta^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_\zeta^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_\zeta^2 \end{bmatrix} \qquad (14)$$

where φ represents a constant autoregressive parameter specifying the relationship between adjacent $\eta_j$ constructs, $\sigma_\eta^2$ is the aforementioned variance of the series, and $\sigma_\zeta^2$ is the variance of the random normal disturbances described in Equation 4. The ARMA(1, 1) process adds a moving average parameter to the **Ψ** matrix, which makes the variance of the series $\sigma_\eta^2 = \frac{(1+\theta)\sigma_\zeta^2}{(1-\varphi)^2}$ (Chatfield, 2004). We started up the ARMA(1, 1) series by specifying the **Ψ** matrix as:

$$\boldsymbol{\Psi} = \begin{bmatrix} \sigma_\eta^2 & \theta\sigma_\zeta^2 & 0 & 0 & 0 \\ \theta\sigma_\zeta^2 & \sigma_\zeta^2(1+\theta^2) & \theta\sigma_\zeta^2 & 0 & 0 \\ 0 & \theta\sigma_\zeta^2 & \sigma_\zeta^2(1+\theta^2) & \theta\sigma_\zeta^2 & 0 \\ 0 & 0 & \theta\sigma_\zeta^2 & \sigma_\zeta^2(1+\theta^2) & \theta\sigma_\zeta^2 \\ 0 & 0 & 0 & \theta\sigma_\zeta^2 & \sigma_\zeta^2(1+\theta^2) \end{bmatrix}. \quad (15)$$

where θ represents the constant moving average parameter that specifies the relationship between adjacent latent errors for occasions $j - 1$ and $j$ and the other terms are as described above.

The three estimating COFMs were fit to the 1,000 data sets for each of the conditions, and the convergence rates and percentages of inadmissible solutions were recorded. Inadmissible solutions were then removed and additional data sets were simulated until 1,000 admissible solutions were obtained for each condition.

*Data Analysis*

The first step in comparing the performance of the three growth models was an examination of the convergence rates. The percentage of non-convergent cases and inadmissible solutions for the first 1,000 datasets generated under each method under each condition was reported. To determine how well parameters were estimated under the different model specifications, relative bias was computed for the point estimates and standard error estimates of the fixed effects. Relative bias was also computed for point estimates of the random effects. The following equation was used to compute relative bias for the point estimates of the fixed and random effects:

$$RB = \frac{\bar{\hat{!}} \quad !}{!}, \quad (?)$$

where $\bar{\hat{!}}$ is the mean parameter estimate and $!$ is the true parameter value.

Hoogland and Boomsma's (1998) criteria for substantial mean relative bias (i.e., when |MRB| > .05) and was adopted. When the true parameter value is 0 (e.g., the covariance between the level and shape), relative bias cannot be calculated. Instead, when the mean absolute values of the simple bias exceeded .05, estimates were considered biased.

A fundamental issue that must be determined prior to interpreting the SEM parameter estimates is whether the model fits the data. Model fit refers to the degree to which the model-implied covariance matrix matches the observed covariance matrix of the variables (Bollen, 1989). In accordance with Leite's (2007) simulation study examining the performance of the COFM, the models in this study were evaluated based on chi-squared statistics and overall fit criteria that indicated acceptable fit.

The three overall fit criteria examined included the comparative fit index (CFI), the Tucker-Lewis Index (TLI), and the root mean squared error of approximation (RMSEA). The goodness of fit (GOF) for each model specification across each condition was evaluated according to the proportion of times the CFI, TLI, and RMSEA indicated acceptable fit following Hu and Bentler's recommendations (1999). According to these recommendations, models can be considered to fit the data well if they produce values greater than or equal to .95 for the CFI and TLI, and values less than or equal to .05 for the RMSEA. The GOF proportion was calculated using an indicator variable, where the variable was assigned a value of one if a particular fit statistic indicated that the model fit the data well in accordance with Hu and Bentler's recommendations, and a zero otherwise. We also examined the statistical non-significance of the chi-squared statistic as an indicator of good model fit.

Following the recommendation that simulation studies be analyzed using the same tools as other experimental studies (Hauck & Anderson, 1984), repeated measures factorial (M)ANOVAs were conducted using sample size, series length, and autocorrelation specification as explanatory variables. When comparing the results across models, the estimating model was treated as a repeated measures factor. For all analyses the outcome measures of interest were: the simple bias of the covariance between the level and shape factors (and the level factor in the absence of autocorrelation), and the relative bias of the level (in the presence of autocorrelation) and shape factor means, and of the level and shape factors' variances.

Due to the large number of observations, the partial $\eta^2$, $!^{2}_{P}$, effect size was used to identify practically significant effects rather than statistically significance results. $!^{2}_{P}$ values greater than .01 were considered practically significant. The .01 value was chosen based on practice in previous simulation research (e.g., Krull & MacKinnon, 1999), and because .01 is the cutoff for a "small" effect (Cohen, 1988; Olejnik & Algina, 2000).

## Results

We compared the three models with respect to convergence rates and proportions of inadmissible solutions, bias in the estimation of parameters and standard errors, and performance of fit criteria in terms of identifying the correct model. Some of the tabled results presented in this section are collapsed across conditions to facilitate interpretation.

*Convergence and Proportion of Inadmissible Solutions*

The proportions of inadmissible solutions presented in Table 1 are collapsed across the series length conditions to facilitate interpretation. All of the models converged across all conditions; however, there were differences in the number of inadmissible solutions produced by the different models. The COFM, which performed better than the two models that incorporated autocorrelation parameters, produced zero inadmissible solutions across most conditions. The COFM results were therefore omitted from Table 1.

By contrast, the AR(1) and ARMA(1, 1) models produced substantial numbers of inadmissible solutions across many conditions. The inadmissible solutions occurred because the variance/covariance matrix of the level and shape was non-positive definite. A *p* x *p* matrix can be defined as non-positive definite if some of the matrix's *p* eigenvalues are less than zero (Wothke, 1993). In particular, negative estimates of the α and β variance parameters were commonplace under certain conditions. In general, the ARMA(1, 1) produced fewer inadmissible solutions than the AR(1) model, and both models produced fewer inadmissible solutions as the sample size and series length increased.

*Fixed Effects*

The relative bias estimates for the intercept and slope parameters reported in Tables 2 and 3 refer to estimates of the intercept (i.e., $\mu_\delta$) and slope (i.e., $\mu_\gamma$), which depend on estimates of $\mu_\alpha$, $\mu_\beta$, and φ as described in Equations 9 and 10.

ANOVA results indicated no significant differences in the relative bias of estimates of the intercept and slope across conditions. As presented in Tables 2 and 3, the intercept and slope were generally estimated accurately across conditions. One exception to this general pattern occurred under the ARMA(1, 1) generating condition where $\varphi = .8$ and $\theta = .3$, the series length was 8, and the estimating model was AR(1) whereupon estimates of the intercept and slope were significantly biased (see Tables 2 and 3).

*Random Effects*

The relative bias estimates for the variance components reported in Tables 4 – 6 refer to estimates of the intercept variance (i.e., $\sigma_\delta^2$) and slope variance (i.e., $\sigma_\gamma^2$), and the covariance between the intercept and slope (i.e., $\sigma_{\gamma\gamma}$ which depend on estimates of $\sigma_\alpha^2$, $\sigma_\beta^2$, $\sigma_{\alpha\beta}$, and $\varphi$ as described in Equations 11 and 12.

Relative bias in estimates of the variance of the intercept depended on the specified model ($\eta_P^2$ = .66) and two three-way interactions: one interaction among estimating model, sample size, and series length ($\eta_P^2$ = .01), and the other among model, series length, and autocorrelation magnitude ($\eta_P^2$ = .33). Intercept variance estimates were considered to be biased across all conditions where the autocorrelation magnitude was large (i.e., $\varphi$ = .8.). In general, estimation was best under the COFM when the magnitude of autocorrelation present in the data was small to moderate (i.e., $\varphi \leq .5$) and the series length was 8 (see Table 4). When the autocorrelation magnitude was large, estimation tended to be best when the estimating model matched the generating model and the series length was large.

Relative bias in the estimates of the variance of the slope depended on the model specification ($\eta_P^2$ = .87) and two three-way interactions: one interaction among model estimated, sample size, and series length ($\eta_P^2$ = .01), and the other among model, series length, and autocorrelation magnitude ($\eta_P^2$ = .34). The bias trends of the slope variance estimates were similar to those of the intercept variance estimates (see Table 5). In general, estimation was best when sample size and series length were large and the magnitude of autocorrelation present in the data was small to moderate.

Bias in the estimation of the covariance between the intercept and slope was influenced by three-way interaction effects among the estimating model, sample size, and autocorrelation magnitude ($\eta_P^2$ = .45). The COFM tended to underestimate the covariance between the intercept and slope, whereas the AR(1) and ARMA(1, 1) models overestimated the covariance under conditions where $\varphi \leq .5$ but underestimated the covariance under conditions where $\varphi = .8$. In general, estimation was best when sample size and series length were large and the estimating model matched the generating model.

*Fit Criteria*

The models were evaluated as to how well they fit the generated data across conditions by the CFI, TLI, RMSEA and chi-squared statistics. Because the pattern of results for the RMSEA closely mirrored those of the CFI and TLI, only the RMSEA results are presented (see Table 1). Where differences existed, the RMSEA was slightly more discriminating than the other two criteria. For example, the TLI and CFI indicated good model fit for every model across each condition with a sample size of at least 200, and the GOF proportion was greater than .94 for every model across each condition when the sample size was 100. In other words, the TLI and CFI almost always indicated that each of the models fit the data well across every condition[1].

The AR(1) and ARMA(1, 1) models were consistently supported by the RMSEA as fitting the data well across each condition. The COFM, in contrast, was indicated as fitting the data well across most but not all conditions by the RMSEA (e.g., AR (1) where φ = .8). The $\chi^2$ statistic, as expected, was more discriminating than the CFI, TLI, and RMSEA. When sample size and series length were small, the GOF proportions for all of the models were generally high (i.e., higher than .700); however, the GOF proportion approached zero for the conditions when the under-specified COFM was estimated as sample size and series length increased under non-zero autocorrelation conditions.

By contrast, the chi-squared statistic indicated that the AR(1) and ARMA(1, 1) models had adequate fit more consistently across conditions. The GOF proportions as indicated by the $\chi^2$ statistic for the two models were similar when series length was small; however, when series length was large, the ARMA(1, 1) model was indicated by the $\chi^2$ statistic as fitting the data well more often than the AR(1) model, particularly under the ARMA(1, 1) conditions (i.e., when the ARMA(1, 1) model was correctly specified and the AR(1) model was misspecified).

## Discussion

This study was motivated by two overarching goals: the first was to examine the effects of autocorrelation on growth parameter estimates of interest under the curve-of-factors growth model; the second was an attempt to modify the COFM to measure growth and autocorrelation processes within the same data set. The study builds upon and was inspired by the research of Sivo et al. (2005), which found that unmodeled autocorrelation resulted in biased growth parameter estimates under a first-order latent growth model, and by Curran and Bollen (2001) who developed a first-order ALT model designed to measure autocorrelation and growth within the same data set.

The results indicate that the COFM is able to produce unbiased estimates of the parameters of interest when the series length is long and the magnitude of autocorrelation present in the data is small to moderate. This study also adds to findings from the body of research indicating that growth models tend to estimate the variance components poorly in the presence of autocorrelation as the magnitude of the autocorrelation in the data increases. Because the true structures of the model's covariance matrices rarely are known in practice, fit criteria are usually used to gauge whether or not the discrepancy between the true composite covariance matrix and the model-implied covariance matrix is acceptable.

Using Hu and Bentler's (1999) suggested criteria of CFI $\geq 0.95$, TLI $\geq 0.95$, and RMSEA $\leq 0.05$ to gauge acceptable model fit, the COFM would have been retained as fitting the data well under most conditions evaluated in this study, even under conditions under which the variance components were estimated poorly. Although there were small sample size conditions under which the model would have been rejected at a high rate [e.g., ARMA (1, 1) where $\varphi = .5$ and $\theta = -.3$], the model would have been evaluated as fitting the data well at least 95% of the time with series length of 5 and a sample size of at least 500. Researchers should therefore be cautious when drawing inferences about the variance components under the COFM if there is reason to believe that autocorrelation is present in the data.

Because previous research has indicated that unmodeled autocorrelation can cause growth models to produce biased estimates of parameters of interest, the second goal of this study was an attempt to measure both growth and autocorrelation processes within the same data set. The results of this study indicate that the fixed effects were estimated well by the models combining growth and autocorrelation parameters across the examined conditions, although there was some evidence of bias when the generating model was ARMA(1, 1) and the estimating model was AR(1). In general, the variance components were not estimated well; however, estimates improved as sample size and series length increased, suggesting that sufficient sample size and series length may resolve this issue.

An aspect of the performance of the AR(1) and ARMA(1, 1) models that was worse than expected was the number of inadmissible solutions produced by each model. The number of inadmissible solutions produced in this study were higher under the AR(1) and ARMA(1, 1) models than under the COFM across all conditions. There are several possible factors that can increase the probability that a matrix of variance/covariance estimates will be non-positive definite. First, the probability of having a non-positive definite solution is higher if the sample size and/or the number of indicators is small (Boomsma, 1985). To test whether or not either of these factors would affect the number of inadmissible solutions, we simulated 1,000 additional data sets under two conditions: the AR(1) condition where $\varphi = .3$ and the ARMA (1, 1) where $\varphi = .5$ and $\theta = -.3$. The series length was 8 for both. We chose to examine these particular conditions because they produced the largest proportions of inadmissible solutions across sample sizes for both the AR(1) and ARMA(1, 1) data generating models.

Although the small number of indicators per factor did not appear to contribute to the number of inadmissible solutions, the sample size did appear to be a contributing factor, as the number of inadmissible solutions decreased as sample size increased across conditions. Increasing the sample size led to a decrease in the number of inadmissible solutions for the ARMA(1, 1) model such that no inadmissible solutions were produced under either condition when the sample size was at least 5,000. The results for the AR(1) model were not as straightforward. The number of inadmissible solutions also continued to decline under the AR(1) condition where $\varphi = .3$, though 15% of the solutions were inadmissible even with a sample size of 10,000. The number of inadmissible solutions produced by the AR(1) model did not decrease under the ARMA(1, 1) condition, when the model was misspecified.

To summarize, although the COFM with autocorrelation parameters performed poorly under many conditions in this study, it appears that increasing sample size and series length sufficiently may ameliorate most issues.

Across all of the models investigated, the factor that most strongly influenced the relative bias of the estimates was the magnitude of the autocorrelation present in the data. In the absence of autocorrelation, all of the methods tested produced unbiased parameter estimates of the fixed effects and variance components under the combination of large sample size and series length. When the autoregressive parameter $\varphi = .8$, variance components were poorly estimated across all conditions, even when the model was specified correctly. This poses potential problems for applied researchers in the social sciences, as it seems reasonable that growth and autocorrelation could be present in the same data set when subjects are measured repeatedly with the same instrument.

*Limitations and Suggestions for Future Research*

The data simulation and analyses conducted in the study have provided some indication of the influence of autocorrelation on the estimation of growth parameters under the COFM. However, a simulation study by design offers a limited set of conditions, and therefore omits other conditions that could produce interesting or contradictory results. For example, an examination of the performance of the ARMA(1, 1) model while increasing the sample size substantially may be worthy of future investigation.

One limitation of this study is that it only examined conditions where the measurement model was fixed across conditions. In particular, each factor was indicated by four observed variables, the item parameters were parallel within a factor, and strict factorial invariance was observed across measurement occasions. Future research could examine the impact of autocorrelation on growth while varying the number of manifest indicators, the item parameters (e.g., tau-equivalent or congeneric), and invariance conditions (e.g., weak or configural).

Another limitation of this study is that only linear growth was simulated. Including curvilinear growth would have extended the simulation beyond manageable proportions; however, an examination of the effects of autocorrelation on non-linear functional forms of growth may be useful. Finally, it is possible that there are alternative specifications for these models that would enable better performance. More research into the causes of the biased estimates of the standard errors of the fixed effects in this study may be warranted.

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## Appendix

This proof generalizes Hamaker's (2005) demonstration of the algebraic equivalence of the ALT model and Latent Growth Curve model with autocorrelated errors, when the autoregressive parameter φ is invariant across time and $|\varphi| < 1$, to the case where an invariant moving average parameter θ is added to the model ($|\theta| < 1$). Adding the moving average parameter θ to the ALT model results in:

$$y_{j,i} = \alpha_i + j\beta_i + \varphi y_{j-1,i} + \theta\epsilon_{j-1,i} + \epsilon_{j,i}, \tag{A1}$$

where $y_{j,i}$ is the dependent variable observed at time *j* for individual *i*, $\alpha_i$ is a constant, $\beta_i$ is the regression coefficient by which the current *y* is regressed on time, φ is the autoregressive parameter by which the current *y* is regressed on the previous *y*, $\epsilon_{j,i}$ are random normally distributed residuals, and θ is the moving average parameter by which the current y is regressed on the previous $\epsilon_{j,i}$.

First, we recursively enter the expression of the previous observation into Equation 1:

$$y_{j,i} = \alpha_i + j\beta_i + \varphi y_{j-1,i} + \theta\epsilon_{j-1,i} + \epsilon_{j,i}$$

$$= \alpha_i + j\beta_i + \varphi\left(\alpha_i + (j-1)\beta_i + \varphi y_{j-2,i} + \theta\epsilon_{j-2,i} + \epsilon_{j-1,i}\right) + \theta\epsilon_{j-1,i} + \epsilon_{j,i}$$

$$= \alpha_i + \varphi\alpha_i + j\beta_i + (j-1)\varphi\beta_i + \varphi^2 y_{j-2,i} + \varphi\theta\epsilon_{j-2,i} + \theta\epsilon_{j-1,i} + \varphi\epsilon_{j-1,i} + \epsilon_{j,i}$$

$$= \alpha_i + \varphi\alpha_i + j\beta_i + (j-1)\varphi\beta_i$$

$$+ \varphi^2\left(\alpha_i + (j-2)\beta_i + \varphi y_{j-3,i} + \theta\epsilon_{j-3,i} + \epsilon_{j-2,i}\right) + \varphi\theta\epsilon_{j-2,i} + \theta\epsilon_{j-1,i} + \varphi\epsilon_{j-1,i} + \epsilon_{j,i}$$

$$= \alpha_i + \varphi\alpha_i + \varphi^2\alpha_i + j\beta_i + (j-1)\varphi\beta_i + (j-2)\varphi^2\beta_i + \varphi^3 y_{j-3,i}$$

$$+ \varphi^2\theta\epsilon_{j-3,} + \varphi\theta\epsilon_{j-2,i} + \theta\epsilon_{j-1,i} + \varphi^2\epsilon_{j-2,i} + \varphi\epsilon_{j-1,i} + \epsilon_{j,i}$$

…

$$= \sum_{n=0}^{\infty} \varphi^n \alpha_i + \sum_{n=0}^{\infty} (j-n) \varphi^n \beta_i + \sum_{n=0}^{\infty} \varphi^n \theta \epsilon_{j-n-1,i} + \sum_{n=0}^{\infty} \varphi^n \epsilon_{j-n,i} \quad \text{(A2)}$$

We can rewrite the last two terms of Equation 2 as

$$z_{j,i} = \varphi z_{j-1,i} + \theta \epsilon_{j-1,i} + \epsilon_{j,i} \ , \quad \text{(A3)}$$

which is an ARMA(1,1) process. The remainder of the proof follows Hamaker's (2005); using the geometric series, Equation 3 simplifies to:

$$y_{j,i} = \sum_{n=0}^{\infty} \varphi^n \alpha_i + \sum_{n=0}^{\infty} (j-n) \varphi^n \beta_i + z_{j,i}$$

$$= \alpha_i \sum_{n=0}^{\infty} \varphi^n + \beta_i \sum_{n=0}^{\infty} (j-n) \varphi^n + z_{j,i}$$

$$= \alpha_i \sum_{n=0}^{\infty} \varphi^n + \beta_i \{\sum_{n=0}^{\infty} j \varphi^n - \sum_{n=0}^{\infty} n \varphi^n\} + z_{j,i}$$

$$= \alpha_i \sum_{n=0}^{\infty} \varphi^n - \beta_i \varphi \sum_{n=0}^{\infty} n \varphi^{n-1} + j \beta_i \sum_{n=0}^{\infty} \varphi^n + z_{j,i}$$

$$= \alpha_i (1-\varphi)^{-1} - \beta_i \varphi (1-\varphi)^{-2} + j \beta_i (1-\varphi)^{-1} + z_{j,i}$$

$$= \delta_i + j \gamma_i + z_{j,i}, \quad \text{(A4)}$$

which can be recognized as a Latent Growth Curve Model with intercept $\delta_i$ and slope $\gamma_i$. The mean and variance of the intercept $\delta_i$ can be expressed as functions of the means and variances of the level $\alpha_i$, shape $\beta_i$, and autoregressive parameter φ:

$$\mu_\delta = \mathrm{E}[\delta_i] = \mu_\alpha (1-\varphi)^{-1} - \varphi \mu_\beta (1-\varphi)^{-2}, \quad \text{(A5)}$$

$$\sigma_\delta^2 = \mathrm{E}[\{\delta_i - \mu_\delta\}^2] = \sigma_\alpha^2 (1-\varphi)^{-2} + \varphi^2 \sigma_\beta^2 (1-\varphi)^{-4} - 2\varphi \sigma_{\alpha\beta} (1-\varphi)^{-3} \quad \text{(A6)}$$

Likewise, the mean and variance of the slope $\gamma_i$ can be written as a function of the mean and variance of the shape $\beta_i$ and the autoregressive parameter φ:

$$\mu_\gamma = \mathrm{E}[\gamma_i] = \mu_\beta (1-\varphi)^{-1}, \quad \text{(A7)}$$

$$\sigma_\gamma^2 = E\left[\{\gamma_i - \mu_\gamma\}^2\right] = \sigma_\beta^2 (1-\varphi)^{-2}. \quad \text{(A8)}$$

Finally, the covariance between $\delta_i$ and $\gamma_i$ can be written as a function of the variances and covariance of $\alpha_i$, shape $\beta_i$, and autoregressive parameter φ:

$$\sigma_{\gamma\delta} = \mathrm{E}\left[\{\gamma_i - \mu_\gamma\}\{\delta_i - \mu_\delta\}\right] = \sigma_{\alpha\beta}(1-\varphi)^{-2} - \varphi\sigma_\beta^2(1-\varphi)^{-3}. \quad \text{(A9)}$$

## Footnotes

[1] Full tables are available from the first author upon request.

Table 1

*Proportion of Inadmissible Solutions for the AR(1) and ARMA(1, 1) COFMs Collapsed across Series Length Conditions*

| | | Data Generating Model | | | | |
|---|---|---|---|---|---|---|
| Estimating Model | N | Zero Autocorrelation | AR(1) (φ = .3) | AR(1) (φ = .8) | ARMA(1, 1) (φ = .8, θ = .3) | ARMA(1, 1) (φ = .5, θ = -.3) |
| AR(1) | 100 | 0.333 | 0.437 | 0.127 | 0.144 | 0.317 |
| | 200 | 0.190 | 0.321 | 0.065 | 0.137 | 0.262 |
| | 500 | 0.096 | 0.195 | 0.038 | 0.111 | 0.240 |
| | 1000 | 0.067 | 0.143 | 0.013 | 0.083 | 0.220 |
| ARMA(1, 1) | 100 | 0.199 | 0.269 | 0.139 | 0.053 | 0.274 |
| | 200 | 0.149 | 0.200 | 0.067 | 0.016 | 0.171 |
| | 500 | 0.082 | 0.118 | 0.034 | 0.003 | 0.065 |
| | 1000 | 0.033 | 0.063 | 0.021 | 0.000 | 0.027 |

*Note*. N = sample size. Proportions based on 1,000 replications.

Table 2

*Mean Relative Bias[a] of the Intercept*

| Estimating Model | L | N | Data Generating Model: Zero Autocorrelation | AR(1) (φ = .3) | AR(1) (φ = .8) | ARMA(1, 1) (φ = .8, θ = .3) | ARMA(1, 1) (φ = .5, θ = -.3) |
|---|---|---|---|---|---|---|---|
| COFM | 5 | 100 | 0.003 | 0.001 | -0.002 | 0.007 | -0.001 |
| | | 200 | -0.004 | 0.004 | 0.001 | 0.004 | -0.001 |
| | | 500 | 0.001 | -0.001 | -0.004 | < 0.001 | 0.002 |
| | | 1000 | < 0.001 | 0.003 | 0.002 | < 0.001 | 0.001 |
| | 8 | 100 | 0.001 | 0.004 | 0.009 | -0.001 | -0.003 |
| | | 200 | < 0.001 | < 0.001 | -0.002 | -0.001 | -0.006 |
| | | 500 | 0.001 | < 0.001 | 0.001 | -0.005 | -0.006 |
| | | 1000 | -0.001 | -0.002 | < 0.001 | 0.004 | -0.001 |
| AR(1) | 5 | 100 | -0.025 | 0.001 | 0.009 | 0.008 | 0.029 |
| | | 200 | -0.010 | 0.012 | -0.007 | < 0.001 | -0.003 |
| | | 500 | -0.005 | 0.001 | 0.008 | -0.002 | -0.049 |
| | | 1000 | -0.002 | < 0.001 | -0.003 | < 0.001 | -0.034 |
| | 8 | 100 | 0.017 | 0.022 | 0.039 | **-0.356** | **0.096** |
| | | 200 | 0.004 | 0.001 | 0.025 | **-0.342** | **0.085** |
| | | 500 | 0.006 | -0.004 | 0.022 | **-0.325** | **-0.051** |
| | | 1000 | 0.003 | 0.001 | 0.002 | **-0.329** | **-0.103** |
| ARMA(1, 1) | 5 | 100 | **-0.050** | -0.016 | 0.044 | -0.003 | 0.020 |
| | | 200 | -0.021 | 0.003 | 0.005 | 0.004 | -0.014 |
| | | 500 | -0.014 | -0.003 | -0.015 | 0.002 | 0.005 |
| | | 1000 | -0.001 | 0.001 | 0.006 | -0.003 | -0.009 |
| | 8 | 100 | -0.024 | -0.003 | 0.004 | 0.012 | 0.012 |
| | | 200 | -0.003 | 0.001 | 0.003 | 0.026 | -0.008 |
| | | 500 | 0.003 | -0.003 | 0.008 | 0.004 | 0.009 |
| | | 1000 | 0.002 | < 0.001 | -0.010 | 0.011 | < 0.001 |

*Note*. L = series length. N = sample size. Estimates based on 1,000 replications. Bolded values represent parameter estimates that were considered to be substantially biased.

[a]Simple bias was computed for the zero autocorrelation condition, because the true parameter value of the level was zero.

Table 3

*Mean Relative Bias of the Slope*

| Estimating Model | L | N | Zero Autocorrelation | AR(1) (φ = .3) | AR(1) (φ = .8) | ARMA(1, 1) (φ = .8, θ = .3) | ARMA(1, 1) (φ = .5, θ = -.3) |
|---|---|---|---|---|---|---|---|
| CoFM | 5 | 100 | < 0.001 | 0.004 | -0.001 | -0.013 | -0.002 |
| | | 200 | 0.004 | 0.002 | 0.005 | 0.005 | 0.007 |
| | | 500 | < 0.001 | 0.001 | -0.003 | 0.002 | -0.001 |
| | | 1000 | -0.001 | -0.002 | < 0.001 | -0.002 | -0.002 |
| | 8 | 100 | -0.001 | -0.001 | 0.003 | -0.004 | 0.002 |
| | | 200 | < 0.001 | -0.001 | -0.004 | 0.001 | < 0.001 |
| | | 500 | 0.001 | < 0.001 | < 0.001 | < 0.001 | 0.001 |
| | | 1000 | 0.001 | 0.001 | -0.004 | 0.005 | < 0.001 |
| AR(1) | 5 | 100 | 0.050 | 0.001 | -0.021 | -0.004 | -0.012 |
| | | 200 | 0.028 | -0.006 | -0.002 | -0.006 | -0.002 |
| | | 500 | 0.007 | -0.004 | < 0.001 | -0.001 | 0.015 |
| | | 1000 | -0.001 | 0.001 | 0.004 | 0.001 | 0.011 |
| | 8 | 100 | < 0.001 | -0.005 | -0.020 | **0.108** | -0.021 |
| | | 200 | < 0.001 | -0.002 | -0.014 | **0.104** | -0.010 |
| | | 500 | -0.006 | -0.001 | -0.013 | **0.099** | 0.005 |
| | | 1000 | -0.004 | -0.002 | -0.001 | **0.103** | 0.008 |
| ARMA(1, 1) | 5 | 100 | **0.094** | 0.009 | -0.016 | -0.001 | -0.010 |
| | | 200 | 0.043 | -0.002 | -0.004 | 0.004 | 0.011 |
| | | 500 | 0.022 | 0.001 | 0.004 | < 0.001 | -0.002 |
| | | 1000 | 0.004 | < 0.001 | -0.006 | 0.001 | 0.007 |
| | 8 | 100 | 0.037 | 0.001 | -0.003 | 0.008 | -0.016 |
| | | 200 | 0.004 | 0.004 | -0.005 | -0.010 | 0.010 |
| | | 500 | -0.002 | 0.003 | -0.005 | -0.004 | -0.005 |
| | | 1000 | -0.004 | < 0.001 | 0.006 | -0.006 | 0.001 |

Data Generating Model: columns Zero Autocorrelation through ARMA(1, 1) (φ = .5, θ = -.3).

*Note*. L = series length. N = sample size. Estimates based on 1,000 replications. Bolded values represent parameter estimates that were considered to be substantially biased.

Table 4

*Mean Relative Bias of the Variance of the Intercept*

| Estimating Model | L | N | Data Generating Model: Zero Autocorrelation | AR(1) (φ = .3) | AR(1) (φ = .8) | ARMA(1, 1) (φ = .8, θ = .3) | ARMA(1, 1) (φ = .5, θ = -.3) |
|---|---|---|---|---|---|---|---|
| CoFM | 5 | 100 | -0.004 | -0.036 | **-0.940** | **-0.961** | **-0.154** |
| | | 200 | < 0.001 | -0.021 | **-0.941** | **-0.961** | **-0.147** |
| | | 500 | -0.001 | -0.015 | **-0.940** | **-0.961** | **-0.154** |
| | | 1000 | -0.007 | -0.015 | **-0.940** | **-0.961** | **-0.148** |
| | 8 | 100 | 0.001 | -0.024 | **-0.921** | **-0.945** | -0.027 |
| | | 200 | -0.003 | -0.016 | **-0.922** | **-0.944** | -0.028 |
| | | 500 | -0.015 | -0.016 | **-0.922** | **-0.944** | -0.031 |
| | | 1000 | -0.001 | -0.019 | **-0.922** | **-0.944** | -0.028 |
| AR(1) | 5 | 100 | **0.655** | **0.603** | **-0.719** | **-0.907** | **10.634** |
| | | 200 | **0.345** | **0.360** | **-0.634** | **-0.920** | **10.672** |
| | | 500 | **0.083** | **0.132** | **-0.476** | **-0.924** | **11.785** |
| | | 1000 | 0.017 | **0.050** | **-0.332** | **-0.926** | **11.306** |
| | 8 | 100 | **-0.072** | -0.044 | **-0.447** | **-0.825** | **1.858** |
| | | 200 | **-0.064** | **-0.067** | **-0.513** | **-0.835** | **1.039** |
| | | 500 | **-0.066** | -0.043 | **-0.568** | **-0.837** | **0.330** |
| | | 1000 | **-0.062** | -0.032 | **-0.598** | **-0.839** | **-0.080** |
| ARMA(1, 1) | 5 | 100 | **1.112** | **0.772** | **-0.740** | **-0.762** | **1.710** |
| | | 200 | **0.679** | **0.602** | **-0.671** | **-0.752** | **1.163** |
| | | 500 | **0.220** | **0.317** | **-0.566** | **-0.763** | **0.877** |
| | | 1000 | **0.089** | **0.158** | **-0.526** | **-0.759** | **0.592** |
| | 8 | 100 | **0.178** | **0.140** | **-0.506** | **-0.475** | **0.436** |
| | | 200 | **0.078** | **0.087** | **-0.547** | **-0.497** | **0.238** |
| | | 500 | 0.011 | **0.072** | **-0.601** | **-0.525** | **0.054** |
| | | 1000 | 0.013 | 0.037 | **-0.613** | **-0.532** | -0.003 |

*Note*. L = series length. N = sample size. Estimates based on 1,000 replications. Bolded values represent parameter estimates that were considered to be substantially biased.

Table 5

*Mean Relative Bias of the Variance of the Slope*

| Estimating Model | L | N | Data Generating Model: Zero Autocorrelation | AR(1) (φ = .3) | AR(1) (φ = .8) | ARMA(1, 1) (φ = .8, θ = .3) | ARMA(1, 1) (φ = .5, θ = -.3) |
|---|---|---|---|---|---|---|---|
| CoFM | 5 | 100 | 0.002 | **0.103** | **-0.701** | **-0.688** | **0.182** |
| | | 200 | -0.008 | **0.099** | **-0.703** | **-0.691** | **0.184** |
| | | 500 | -0.004 | **0.102** | **-0.701** | **-0.689** | **0.184** |
| | | 1000 | -0.010 | **0.098** | **-0.702** | **-0.689** | **0.184** |
| | 8 | 100 | -0.013 | -0.003 | **-0.616** | **-0.613** | -0.037 |
| | | 200 | -0.002 | -0.003 | **-0.616** | **-0.609** | -0.029 |
| | | 500 | -0.001 | < 0.001 | **-0.617** | **-0.609** | -0.028 |
| | | 1000 | 0.001 | -0.003 | **-0.616** | **-0.608** | -0.028 |
| AR(1) | 5 | 100 | **0.419** | **0.315** | **-0.300** | **-0.517** | **3.204** |
| | | 200 | **0.208** | **0.209** | **-0.226** | **-0.569** | **2.731** |
| | | 500 | **0.053** | **0.069** | **-0.114** | **-0.592** | **2.635** |
| | | 1000 | 0.003 | 0.022 | -0.044 | **-0.602** | **2.480** |
| | 8 | 100 | -0.002 | 0.025 | **-0.152** | **-0.424** | **0.396** |
| | | 200 | -0.001 | 0.010 | **-0.202** | **-0.446** | **0.260** |
| | | 500 | -0.022 | 0.002 | **-0.250** | **-0.456** | **0.155** |
| | | 1000 | -0.017 | -0.004 | **-0.273** | **-0.461** | **0.120** |
| ARMA(1, 1) | 5 | 100 | **0.995** | **0.604** | **-0.318** | **-0.411** | **1.040** |
| | | 200 | **0.604** | **0.453** | **-0.249** | **-0.423** | **0.699** |
| | | 500 | **0.227** | **0.236** | **-0.174** | **-0.460** | **0.529** |
| | | 1000 | **0.103** | **0.127** | **-0.169** | **-0.467** | **0.368** |
| | 8 | 100 | **0.250** | **0.097** | **-0.175** | **-0.212** | **0.126** |
| | | 200 | **0.111** | **0.067** | **-0.218** | **-0.251** | **0.085** |
| | | 500 | 0.028 | 0.033 | **-0.269** | **-0.280** | 0.040 |
| | | 1000 | 0.009 | 0.021 | **-0.281** | **-0.285** | 0.020 |

*Note*. L = series length. N = sample size. Estimates based on 1,000 replications. Bolded values represent parameter estimates that were considered to be substantially biased.

Table 6

*Mean Simple Bias of the Covariance Between the Intercept and Slope*

| Estimating Model | L | N | Data Generating Model: Zero Autocorrelation | AR(1) (φ = .3) | AR(1) (φ = .8) | ARMA(1, 1) (φ = .8, θ = .3) | ARMA(1, 1) (φ = .5, θ = -.3) |
|---|---|---|---|---|---|---|---|
| CoFM | 5 | 100 | < 0.001 | **-0.155** | **-0.993** | **-1.028** | **-0.489** |
| | | 200 | 0.004 | **-0.121** | **-0.993** | **-1.029** | **-0.477** |
| | | 500 | 0.002 | **-0.113** | **-0.992** | **-1.029** | **-0.493** |
| | | 1000 | 0.002 | **-0.117** | **-0.993** | **-1.029** | **-0.489** |
| | 8 | 100 | 0.001 | **-0.232** | **-0.945** | **-0.983** | **-0.308** |
| | | 200 | 0.002 | **-0.240** | **-0.945** | **-0.981** | **-0.306** |
| | | 500 | 0.001 | **-0.210** | **-0.945** | **-0.982** | **-0.306** |
| | | 1000 | < 0.001 | **-0.209** | **-0.945** | **-0.981** | **-0.304** |
| AR(1) | 5 | 100 | **-0.089** | **1.972** | **-0.673** | **-0.911** | **14.162** |
| | | 200 | -0.047 | **1.229** | **-0.570** | **-0.932** | **13.393** |
| | | 500 | -0.012 | **0.425** | **-0.395** | **-0.940** | **14.267** |
| | | 1000 | -0.003 | **0.154** | **-0.258** | **-0.944** | **13.762** |
| | 8 | 100 | 0.005 | **0.027** | **-0.371** | **-0.805** | **2.170** |
| | | 200 | 0.006 | **-0.056** | **-0.433** | **-0.818** | **1.458** |
| | | 500 | 0.005 | -0.040 | **-0.492** | **-0.822** | **0.923** |
| | | 1000 | 0.005 | -0.030 | **-0.522** | **-0.825** | **0.673** |
| ARMA(1, 1) | 5 | 100 | **-0.177** | **2.924** | **-0.697** | **-0.783** | **2.857** |
| | | 200 | **-0.108** | **2.245** | **-0.607** | **-0.774** | **1.950** |
| | | 500 | -0.034 | **1.160** | **-0.486** | **-0.791** | **1.571** |
| | | 1000 | -0.016 | **0.606** | **-0.446** | **-0.789** | **1.124** |
| | 8 | 100 | -0.026 | **0.455** | **-0.423** | **-0.489** | **0.385** |
| | | 200 | -0.011 | **0.269** | **-0.467** | **-0.518** | **0.253** |
| | | 500 | -0.003 | **0.181** | **-0.525** | **-0.546** | **0.069** |
| | | 1000 | -0.002 | **0.132** | **-0.538** | **-0.553** | -0.005 |

*Note*. L = series length. N = sample size. Estimates based on 1,000 replications. Bolded values represent parameter estimates that were considered to be substantially biased.

Table 9

*Proportion of Replications for which RMSEA Supported Good Model Fit*[a]

| Estimating Model | L | N | Data Generating Model: Zero Autocorrelation | AR(1) (φ = .3) | AR(1) (φ = .8) | ARMA(1, 1) (φ = .8, θ = .3) | ARMA(1, 1) (φ = .5, θ = -.3) |
|---|---|---|---|---|---|---|---|
| CoFM | 5 | 100 | 0.918 | 0.908 | 0.624 | 0.842 | 0.616 |
| | | 200 | 1.000 | 1.000 | 0.910 | 0.999 | 0.909 |
| | | 500 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 |
| | | 1000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 8 | 100 | 0.945 | 0.810 | 0.027 | 0.167 | 0.124 |
| | | 200 | 1.000 | 1.000 | 0.086 | 0.640 | 0.643 |
| | | 500 | 1.000 | 1.000 | 0.061 | 0.970 | 0.973 |
| | | 1000 | 1.000 | 1.000 | 0.018 | 0.995 | 1.000 |
| AR(1) | 5 | 100 | 0.931 | 0.932 | 0.921 | 0.934 | 0.933 |
| | | 200 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 500 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 1000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 8 | 100 | 0.939 | 0.925 | 0.942 | 0.906 | 0.931 |
| | | 200 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 500 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 1000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| ARMA(1, 1) | 5 | 100 | 0.936 | 0.927 | 0.934 | 0.935 | 0.919 |
| | | 200 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 |
| | | 500 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 1000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 8 | 100 | 0.932 | 0.918 | 0.942 | 0.939 | 0.934 |
| | | 200 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 500 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 1000 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 |

*Note*. L = series length. N = sample size. Estimates based on 1,000 replications.

[a] Hu and Bentler's (1999) criterion was used where good model fit is supported if RMSEA ≤ 0.05.

Table 10

*Proportion of Replications for which $\chi^2$ Statistic Supported Good Model Fit*

| Estimating Model | L | N | Data Generating Model: Zero Autocorrelation | AR(1) (φ = .3) | AR(1) (φ = .8) | ARMA(1, 1) (φ = .8, θ = .3) | ARMA(1, 1) (φ = .5, θ = -.3) |
|---|---|---|---|---|---|---|---|
| CoFM | 5 | 100 | 0.793 | 0.734 | 0.379 | 0.634 | 0.353 |
| | | 200 | 0.888 | 0.759 | 0.139 | 0.597 | 0.125 |
| | | 500 | 0.926 | 0.581 | 0.000 | 0.194 | 0.000 |
| | | 1000 | 0.943 | 0.193 | 0.000 | 0.003 | 0.000 |
| | 8 | 100 | 0.312 | 0.141 | 0.000 | 0.002 | 0.003 |
| | | 200 | 0.775 | 0.231 | 0.000 | 0.000 | 0.000 |
| | | 500 | 0.891 | 0.014 | 0.000 | 0.000 | 0.000 |
| | | 1000 | 0.939 | 0.000 | 0.000 | 0.000 | 0.000 |
| AR(1) | 5 | 100 | 0.797 | 0.813 | 0.795 | 0.821 | 0.808 |
| | | 200 | 0.888 | 0.904 | 0.894 | 0.884 | 0.882 |
| | | 500 | 0.928 | 0.939 | 0.948 | 0.910 | 0.895 |
| | | 1000 | 0.947 | 0.949 | 0.934 | 0.896 | 0.875 |
| | 8 | 100 | 0.315 | 0.231 | 0.313 | 0.270 | 0.317 |
| | | 200 | 0.720 | 0.521 | 0.757 | 0.586 | 0.727 |
| | | 500 | 0.855 | 0.488 | 0.901 | 0.539 | 0.838 |
| | | 1000 | 0.875 | 0.308 | 0.930 | 0.233 | 0.813 |
| ARMA(1, 1) | 5 | 100 | 0.811 | 0.790 | 0.811 | 0.802 | 0.797 |
| | | 200 | 0.890 | 0.894 | 0.899 | 0.905 | 0.895 |
| | | 500 | 0.929 | 0.935 | 0.939 | 0.926 | 0.943 |
| | | 1000 | 0.935 | 0.939 | 0.945 | 0.945 | 0.933 |
| | 8 | 100 | 0.360 | 0.300 | 0.354 | 0.329 | 0.334 |
| | | 200 | 0.745 | 0.659 | 0.734 | 0.761 | 0.753 |
| | | 500 | 0.915 | 0.707 | 0.897 | 0.899 | 0.900 |
| | | 1000 | 0.949 | 0.507 | 0.930 | 0.910 | 0.930 |

*Note*. L = series length. N = sample size. Estimates based on 1,000 replications.